arXiv:hep-ph/9607205v1 1 Jul 1996

BUTP–96/15
hep-ph/9607205
June 1996

Comparison of Lattice and Chiral Perturbation Theory calculations of pion scattering lengths

Gilberto Colangelo

Institut für Theoretische Physik
Universität Bern
CH-3012 Bern, Switzerland

Abstract

I compare the lattice calculation of Fukugita et al. for the pion S–wave scattering lengths to the predictions of Chiral Perturbation Theory to two loop accuracy. I find good agreement, despite the use of the quenched approximation in the lattice calculation.

PACS numbers: 12.39.Fe, 12.38.Gc

In this letter I will present the comparison of two results which are already available in the literature: the calculation of the pion S–wave scattering lengths in quenched lattice QCD [1] (see also [2]) and in Chiral Perturbation Theory (CHPT) [3] to two loops [4]. Admittedly, the two results need not be the same: the physical content of the two calculations is different since QCD contains sea quark contributions, whereas the quenched lattice calculation does not. Let me disregard this complication for the moment, I will come to it later.

There is an important reason for attempting a comparison, which however concerns more future developments than present day calculations. As it is well known CHPT provides an efficient technique for implementing the constraints of the chiral symmetry of QCD on the Green functions. Remarkably, at low energy this is enough to make predictions. This approach uses only the symmetry properties of QCD: the dynamics of quarks and gluons is totally ignored and shows up only in the values of some unknown low energy constants, which we usually learn from experiments. The major challenge for bridging the gap between QCD and its low energy effective theory is to reproduce the value of these low energy constants starting from the QCD Lagrangian. The best candidate for doing this is lattice QCD, although very little has been done up to know – the only attempt in this direction of which I am aware is the work by Myint and Rebbi [5].

The comparison of lattice QCD and CHPT for quantities like the pion S–wave scattering lengths is very interesting from this point of view: finding agreement means that we understand the value of these low energy constants – as extracted from experiments – on the basis of the QCD Lagrangian. Of course the full accomplishment of this program requires overcoming the quenched approximation.

The method to calculate scattering lengths on the lattice is due to Lüscher [6]. In principle it allows to extract these quantities in a theoretically very clean manner. The method consists in measuring the energy of the fundamental state of two static particles inside a spatial box of length L. Lüscher has calculated the expansion of this energy in inverse powers of the length L, showing that the first three coefficients of this expansion (which starts with a term of order L^{-3}) contain powers of the S–wave scattering length. From the measurement of the energy shift due to the finite size of the box one can extract the scattering length. The method is very powerful, in particular because it allows to directly relate a calculation done in Euclidean space to a quantity which is defined in Minkowski space. Moreover it transforms what is usually considered a systematic error into an interesting effect from which to extract useful information.

Lüscher's formula can be applied only if $M_\pi L \gg 1$. This condition was satisfied in the calculation of Ref. [1] by using an unphysical, moderately large pion mass. This is not a problem for comparing to the CHPT calculation, since in the chiral expansion the dependence on light quark masses is fully explicit: given $\hat{m} = 1/2(m_u + m_d)$, or

a value for M_π and F_π, one can calculate the corresponding value for the scattering lengths. The only requirement for the chiral expansion to be justified is that the pion mass be small with respect to the typical mass scale of QCD, M_ρ say. This condition is satisfied in the case of staggered fermions, where Fukugita et al. had $M_\pi/M_\rho \sim 0.3$. On the contrary, we cannot compare to the Wilson fermions calculations, where $M_\pi/M_\rho \sim 0.7$.

Let us discuss here in detail how the CHPT expressions have to be calculated in the case of unphysical quark masses. For the purpose of this discussion I will only consider the one loop case. The extension to two loops is straightforward. At one one loop a_0^0 is given by [7]:

$$\begin{aligned}\frac{32\pi F_\pi^2}{M_\pi} a_0^0 &= 7\left\{1+\frac{M_\pi^2}{F_\pi^2}\left[\frac{40}{7}\left(l_1^r(\mu)+l_2^r(\mu)\right)+\frac{10}{7}l_3^r(\mu)+2l_4^r(\mu)\right.\right. \\ &\left.\left. -\frac{9}{32\pi^2}\log\frac{M_\pi^2}{\mu^2}+\frac{5}{32\pi^2}\right]+O\left(\frac{M_\pi^4}{F_\pi^4}\right)\right\} \; . \qquad (1)\end{aligned}$$

This expression is scale independent: the scale dependence of the low energy constants $l_i^r(\mu)$ compensates the one due to the chiral log.

To compare to the lattice result I use the ratio M_π^2/F_π^2 as calculated on the lattice. The $l_i^r(\mu)$ do not depend on the light quark masses and I use for them the values given in Ref. [4]. The last thing to be evaluated is the chiral log. This requires to give the value of M_π in physical units. To make the connection between lattice and physical units I use the ρ mass. This depends on the light quark masses and can be expressed as a power series. Using an ansatz known as the "quark counting rule" (see *e.g.* Ref.[8]), I consider only the first two terms in the expansion, according to:

$$M_\rho = M_V + 2\hat{m} + O(\hat{m}^{3/2}) \; , \qquad (2)$$

where M_V stands for the ρ mass in the chiral limit, for which I use the value $M_V =$ 760 MeV. According to this ansatz, for low values of M_π/M_ρ the M_ρ dependence on the light quark masses makes a rather small effect, and the pion mass in physical units, at $M_\pi/M_\rho = 0.33$ turns out to be $M_\pi = 260$ MeV (using also $B = 2M_V$ [8], where $B \equiv -\langle 0|\bar{q}q|0\rangle/F^2$). It should be noted that the higher the value of the pion mass, the higher is the relative importance of the low energy constants with respect to the chiral log (at fixed μ). This means that on the lattice one can in principle increase the sensitivity of a given quantity to the low energy constants, and improve the accuracy of their determination.

The same procedure has to be repeated in the two loop case. There we have six new constants appearing, the $r_i^r(\mu)$, $i = 1, \ldots 6$ whose value has been estimated via resonance saturation in Ref. [4]. Here I use the same estimate. The effect of these new constants is small.

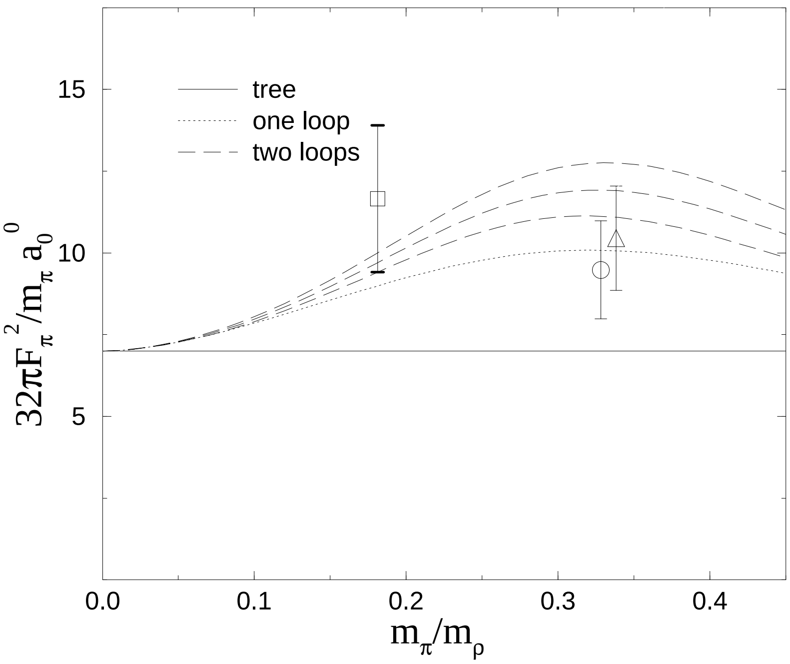

Figure 1: Comparison of the $I = 0$ pion scattering length as calculated on the lattice by Fukugita et al. [1] and the CHPT expansion. The data point with the square is the experimental measurement [9]. The two lattice points are obtained with staggered fermions. The circle refers to the calculation done without gauge fixing, and the triangle to the Coulomb gauge.

The curves for the dimensionless quantities $32F_\pi^2 a_0^I/M_\pi$ as functions of M_π/M_ρ are shown in Figs. 1, 2 for $I = 0, 2$ respectively [11]. The straight solid line corresponds to the Current Algebra (*i.e.* CHPT to tree level) prediction [12], whereas the dotted curve to the one loop CHPT calculation. The three dashed curves are all calculated with the two loop expression for the scattering length. The central one corresponds to the values of the constants used in Ref. [4], whereas the other two are obtained by varying the low energy constant $l_2^r(M_\rho)$ by $\pm 2\ 10^{-3}$: this is a generous error for this particular constant, and is meant to give a rough estimate of the overall uncertainty as coming also from the other low energy constants.

In Figs. 1 and 2 I have shown also the available experimental data points [9, 10], and the lattice points calculated by Fukugita et al. [1]. In the $I = 0$ case the two loop CHPT curve is somewhat higher than the lattice points. In addition, if one looks at the relative contribution of the two loops with respect to the one loop and

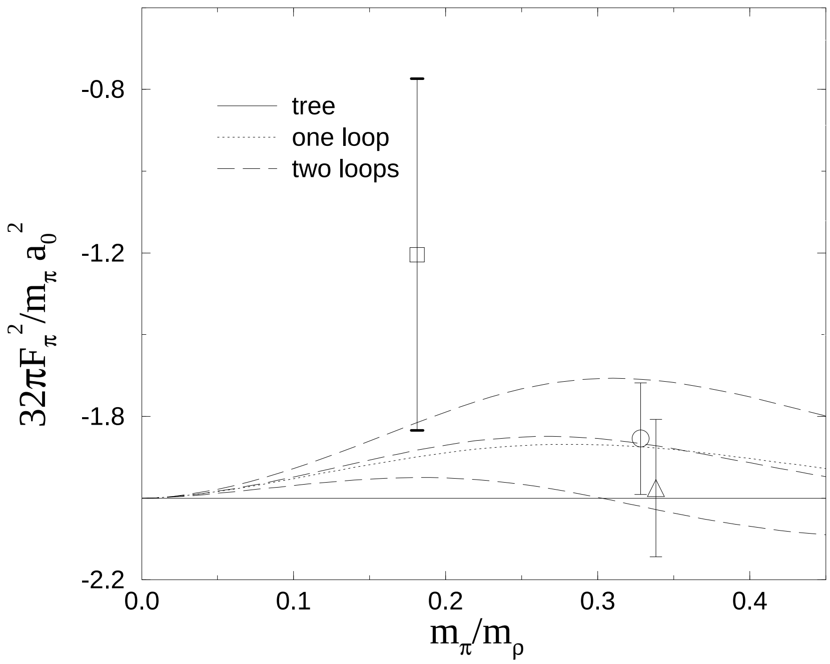

Figure 2: Comparison of the $I = 2$ pion scattering length as calculated on the lattice by Fukugita et al. [1] and the CHPT expansion. The legend for the data points is the same as in Fig. 1; the experimental datum is from Ref. [10]

tree level, one can see that the series is not converging very rapidly, and one may expect positive sizeable contributions from higher orders. In the $I = 2$ case the one and two loop corrections to the Current Algebra value are rather small, and the lattice calculation is in agreement with all of them. All in all, the lattice calculation and CHPT to two loops show deviations from Current Algebra which agree in sign and size surprisingly well, for both isospin cases. This seems to indicate that the effects due to quenching are of the same size, or smaller than the statistical errors of the lattice calculations.

Finally, I come back to the question of a priori estimates of the effect of quenching. In a recent article Bernard and Golterman [13], have analyzed the modifications of Lüscher's formula in the presence of quenching. For this purpose they used the method called quenched CHPT [14]. In this approach one is able to obtain two types of results: first, one can see whether in a specific quantity the quenched approximation removes the chiral logs (in Ref. [14] they showed that in M_π and F_π to one loop there are none); and secondly one can estimate the effects due to the

η–singlet propagator which in this approximation is ill–defined (because it contains a double pole). These results show that the quenched approximation introduces a qualitative change in approaching the chiral limit. For finite quark masses, however, it is difficult to estimate reliably the size of the change: in CHPT the chiral logs come together with the low energy constants that remove the scale dependence (see Eq. (1)), and the splitting between the two is arbitrary. At most one can try to estimate the part of the effect which is due to the double pole in the η–singlet propagator (see Ref. [14] for details). This is what Bernard and Golterman have done in the specific case of Lüscher's formula [13]. Their result is that the quenched approximation modifies considerably the expansion of the energy shift in inverse powers of the box length: in the $I = 0$ case, for example, it introduces terms of order $1/L^n$ with $n = 0, 2$. Nevertheless their numerical estimate of these spurious effects is not too discouraging: for the staggered fermions calculation of Fukugita et al. they estimate a 1% effect in the $I = 0$ case (because of a strong cancellation), and a 20% correction in the $I = 2$ case.

This a priori estimate suggests that the quenched approximation does not spoil completely the results of this lattice calculation, as the comparison proposed here also indicates. On the other hand, the use of quenching certainly diminishes the importance of the agreement with two loop CHPT. In fact, had this result been obtained in full QCD, one could conclude that lattice QCD is able to explain the value of the two combinations of low energy constants appearing in the two scattering lengths: this would be a very remarkable achievement of lattice QCD. Unfortunately, in the present case I cannot go that far, and I have to conclude with the hope that there will be soon various improvements on this lattice calculation. For example, a repetition of this calculation with a different lattice size would allow one to see whether the terms of order $1/L^n$ with $n = 0, 2$ show up, and eventually to remove them. Moreover it would allow a more reliable extraction of the coefficient of the $1/L^3$ term, by explicitly checking the volume dependence. Hopefully, this and other future improvements will tell us whether the agreement observed here contains some real physics, or is only accidental.

Acknowledgements I thank for interesting discussions and/or support J. Gasser, P. Hasenfratz, A. Papa, R. Petronzio and R. Urech , and J. Gasser for very useful comments on the original manuscript. This work has been supported in part by Schweizerischer Nationalfonds and HCM–EEC–Contract No. CHRX–CT920026 (EURODAΦNE).

References

[1] M. Fukugita, Y. Kuramashi, M. Okawa, H. Mino, and A. Ukawa, Phys. Rev. Lett. **71** 2387 (1993); Phys. Rev. D **52** 3003 (1995).

[2] M. Guagnelli, E. Marinari, and G. Parisi, Phys. Lett. B**240** 188 (1990); S. R. Sharpe, R. Gupta, and G.W. Kilcup, Nucl. Phys. B**383** 309 (1992); R. Gupta, A. Patel, and S. R. Sharpe, Phys. Rev. D **48** 388 (1993).

[3] J. Gasser, and H. Leutwyler, Ann. of Phys. (N.Y.) **158** 142 (1984).

[4] J. Bijnens, G. Colangelo, G. Ecker, J. Gasser, and M. Sainio, Phys. Lett. B **374** 210 (1996).

[5] S. Myint, and C. Rebbi, Nucl.Phys. B**421** 241 (1994).

[6] M. Lüscher, Comm. Math. Phys. **105** 153 (1986).

[7] J. Gasser, and H. Leutwyler, Phys. Lett. **125**B 325 (1983).

[8] J. Gasser, and H. Leutwyler, Phys. Rep. C**87** 77 (1982).

[9] L. Rosselet et al., Phys. Rev. D **15** 574 (1977).

[10] M.M. Nagels et al., Nucl. Phys. B**147** 189 (1979).

[11] To obtain the curves I had to vary the ratio M_π/F_π continuously, letting it go through 2.24 at $M_\pi/M_\rho = 0.18$ (physical value), and 4.8 at $M_\pi/M_\rho = 0.33$ (lattice value, see Ref. [1]). How I interpolated between these values is irrelevant for the present discussion.

[12] S. Weinberg, Phys. Rev. Lett. **17** 616 (1966).

[13] C.W. Bernard, and M.F.L. Golterman, Phys. Rev. D **53** 476 (1996).

[14] C.W. Bernard, and M.F.L. Golterman, Phys. Rev. D **46** 853 (1992).